GRAPH THEORY

Tutorial – 5

1) In a large university with k academic departments, we must appoint an important committee. One professor will be chosen from each department. Some professors have joint appointments in two or more departments, but each must be the designated representative of at most one department. We must use equally many assistant professors, associate professors, and full professors among the chosen representatives (assume that k is divisible by 3). How can the committee be found ?

2) Let G be a graph whose odd cycles are pairwise intersecting, meaning that every two odd cycles have a common vertex. Prove that, $\chi(G) \leq 5$.

3) Let X and Y be disjoint sets of vertices in a k-connected graph G. Let u(x) for $x \in X$ and w(y) for $y \in Y$ be non-negative integers such that $\Sigma_{x \in X} u(x) = \Sigma_{y \in Y} w(y) = k$. Prove that, G has k pairwise internally disjoint X,Y-paths so that u(x) of them start at x and w(y) of them end at y, for $x \in X$ and $y \in Y$.

4) Let G be a maximal planar graph. Prove that, G* is 2-edge-connected and 3-regular.

5) Let G be a simple graph with diameter 2, and let [S,S′] be a minimum edge cut with $|S| \leq |S'|$. Prove that, every vertex of S has a neighbor in S′.